Correct usage of quantities, units and equations

Miscellaneous

Brochure | 03.00

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Legal units

More than 100 years ago, the first national laws on electrical units of measurement came into force. This anniversary should remind us that uniform scientific notation is more important than ever at the international level. This brochure gives an overview of electrical quantities and units. For a comprehensive description of quantities and units for all fields of physics, refer to [2].

After the introduction of the International System of Units (abbreviated as SI from the French "Système international d'unités") in 1960, the base units were redefined. Except for a few details of definition, the electrical SI units are identical to the previous electrical units of measurement.

Reference [1] contains the current definitions of the SI units and the standardization of names and letter symbols and their notations. References [3], [4] and [6] define the SI units in accordance with the International System of Quantities (ISQ).

The national standards have been agreed upon with the competent international organizations (ISO and IEC) and describe the internationally recognized state of the art. The SI units have been adopted as the legal units in almost all countries worldwide.

Table 1: ISQ base quantities and SI base units

ISQ base quantity		SI base unit	
Name	**Letter symbol**	**Name**	**Unit symbol**
Length	l	meter	m
Mass	m	kilogram	kg
Time	t	second	s
Electric current	I	ampere	A
Thermodynamic temperature	T, Q	kelvin	K
Amount of substance	n, v	mole	mol
Luminous intensity	I_v	candela	cd

Table 2: Derived electrical quantities and units with special unit symbols [1)]

ISQ quantity		Derived SI unit	
Name	**Letter symbol**	**Name**	**Unit symbol**
Energy	W	joule	J
Power	P	watt	W
Voltage, electric tension	U, V [2)]	volt	V
Electric charge	Q	coulomb	C
Electric capacitance	C	farad	F
Electric resistance	R	ohm	Ω
Electric conductance	G	siemens	S
Magnetic flux	F	weber	Wb
Magnetic flux density	B	tesla	T
Inductance	L	henry	H
Frequency	f	hertz	Hz

1) If it is absolutely clear from the context that quantities of electricity are being referred to, the adjective "electric" can be omitted.

2) U is used in the cited ISO and IEC standards and in most countries; V is used in the USA.

ISQ base quantities and SI base units are listed in Table 1. Derived quantities that are important in electrical engineering and have special units are listed in Table 2. Table 3 contains the prefixes and prefix symbols for decimal submultiples and multiples of units. Prefixes and prefix symbols are exclusively used together with unit names and unit symbols. Prefix symbols and unit symbols are not separated by a space; together, they form the symbol for a new unit.

The concepts of quantity and unit are defined in the International vocabulary of metrology [5].

SI units must be written as stipulated by law or standard and may not be modified by appending additional information such as indices or superscripts or subscripts.

Table 3: Prefixes and prefix symbols for decimal submultiples and multiples of units

Prefix	Symbol	Factor
yocto	y	10^{-24}
zepto	z	10^{-21}
atto	a	10^{-18}
femto	f	10^{-15}
pico	p	10^{-12}
nano	n	10^{-9}
micro	µ	10^{-6}
milli	m	10^{-3}
centi	c	10^{-2}
deci	d	10^{-1}
deca	da	10^{1}
hecto	h	10^{2}
kilo	k	10^{3}
mega	M	10^{6}
giga	G	10^{9}
tera	T	10^{12}
peta	P	10^{15}
exa	E	10^{18}
zetta	Z	10^{21}
yotta	Y	10^{24}

Quantities

Physical phenomena are described qualitatively and quantitatively by physical quantities. Every value of a quantity can be expressed as the product of numerical value and unit. If the unit changes (for example, by adding a prefix symbol), the numerical value changes as well. The product of numerical value and unit remains constant; it is invariant with respect to a change of unit. Example: U = 0.1 V and U = 100 mV describe the same value of quantity.

The letter symbols for physical quantities should consist of only one letter. They are laid down in the international IEC 60027 standards ([7], [8] and [9]). These standards have been adopted by CENELEC as European standards (EN).

If a symbol consists of several letters, it might be misinterpreted in equations as the product of several quantities. Therefore, multiple-letter abbreviations of names should not be used as quantity symbols. When it is necessary to indicate a special meaning of a letter symbol, letters or numerals can be added to the general letter symbol as indices.

Quantities of the same kind are specified in the same unit. They are distinguished either by different letter symbols or by letter symbols with index. Table 4 gives an overview of quantities of the same kind.

Table 4: Examples of quantities of the same kind

Quantity		Unit	
Name	**Letter symbol**	**Name**	**Letter symbol**
Length	l	meter	m
Width	b	meter	m
Height	h	meter	m
Frequency	f	hertz	Hz
Resonance frequency	f_{r}, f_{rsn}	hertz	Hz
Bandwidth	B, f_{B}	hertz	Hz
Voltage, electric tension	U, V	volt	V
RMS value of a voltage	U_{RMS}, V_{RMS}	volt	V
Complex amplitude of a sine voltage	$\underline{\hat{U}}$, $\underline{\hat{V}}$	volt	V
Power	P	watt	W
Signal power	P_{s}	watt	W
Noise power	P_{n}	watt	W
Active power	P, P_{p}	watt	W
Reactive power	Q, P_{q}	watt (also var)	W (also var)
Apparent power	S, P_{s}	watt	W (also VA)

Equations

The terms quantity equation, scaled quantity equation and numerical value equation as well as the relation “value of quantity equals numerical value times unit“ are based on the work of Julius Wallot and others between 1922 and 1933. In 1931, discussions about this topic led to the first edition of the German DIN 1313 standard on the notation of physical equations.

Quantity equations

Quantity equations are equations where the letter symbols represent physical quantities or mathematical symbols ([2], [3], [5] and [6]). These equations are independent of the selected units. When evaluating quantity equations, the products of numerical value and unit must be substituted for the letter symbols. Numerical values and units in quantity equations are treated as independent factors.

Example: The equation

$$U = R \cdot I$$

always yields the same result, irrespective of the units in which resistance R and current I are expressed, provided that the associated products of numerical value and unit are substituted for R and I.

Scaled quantity equations

Scaled quantity equations are quantity equations where every quantity appears with its unit in the denominator.

Example:

$$U/\mathrm{kV} = 10^{-3} \cdot (R/\Omega) \cdot (I/\mathrm{A})$$

The parentheses can be omitted if the assignment of quantities and units is clear without parentheses, for example on the left side of the above equation or when horizontal fraction lines are used:

$$\frac{U}{\mathrm{kV}} = 10^{-3} \cdot \frac{R}{\Omega} \cdot \frac{I}{\mathrm{A}}$$

The advantage of a scaled quantity equation is that the quotients of quantity and unit directly represent the numerical values for the given units. The equations remain correct even if the products of numerical value and unit in other units are substituted for the quantities. In this case, however, the units must be converted. The use of the scaled quantity equation is recommended for representing results.

Numerical value equation

Numerical value equations are equations where the letter symbols represent the numerical values of physical quantities or mathematical symbols ([2], [3], [5] and [6]). These equations are dependent on the selected units.

When coherent units are used, the numerical value equations coincide with the corresponding quantity equations. When identical letter symbols are used for quantities and numerical values, it is not possible to distinguish between a quantity equation and a numerical value equation.

Numerical value equations should no longer be used because they are considered outdated [2]. They must be indicated as numerical value equations, and units must be specified for all quantities.

To indicate the numerical value and the unit of a quantity, the standards use braces or brackets with the following meaning:

$\{U\}$	numerical value of quantity U
$[U]$	unit of quantity U
$U = \{U\} \cdot [U]$	quantity = numerical value · unit

According to the relevant standards, it is not allowed to add units in brackets to quantity symbols in equations or to write the units in brackets in front of, beside or under the equations.

Examples of incorrect usage:

$U\,[\mathrm{kV}] = 10^{-3} \cdot R\,[\Omega] \cdot I\,[\mathrm{A}]$ wrong!

$U = 10^{-3} \cdot R \cdot I$ $U\,[\mathrm{kV}]$, $R\,[\Omega]$, $I\,[\mathrm{A}]$ wrong!

Such notations must never be used. The scaled quantity equation should always be used in order to show the relation between numerical values. The following notations are correct: U in kV, R in Ω, I in A. By the way: These notations are identical in German and English.

Logarithmic ratios of quantities, attenuation and gain figures

Attenuation and gain figures describe the logarithmic ratio of two electrical quantities that identifies the characteristics of a two-port or of a transmission path. The unit used is the decibel (dB). The arguments of the logarithm are numerical values. dB is not an SI unit. Like SI units, dB should not be modified by appending additional information. The function "lg" describes the logarithm to base 10; "log" is the general logarithm function [9].

Definition for power quantities

Example: active power

Power attenuation figure of a two-port:

$$A_P = \left(10\ \mathrm{lg}\frac{P_1}{P_2}\right)\mathrm{dB}$$

Power gain figure of a two-port:

$$G_P = \left(10\ \mathrm{lg}\frac{P_2}{P_1}\right)\mathrm{dB}$$

Definition for quantities whose square is proportional to a power quantity

Example: complex amplitudes or RMS values of alternating voltages

Voltage attenuation figure of a two-port:

$$A_U = \left(20\ \mathrm{lg}\left|\frac{\underline{U}_1}{\underline{U}_2}\right|\right)\mathrm{dB} = \left(20\ \mathrm{lg}\frac{U_{1\,\mathrm{RMS}}}{U_{2\,\mathrm{RMS}}}\right)\mathrm{dB}$$

Voltage gain figure of a two-port:

$$G_U = \left(20\ \mathrm{lg}\left|\frac{\underline{U}_2}{\underline{U}_1}\right|\right)\mathrm{dB} = \left(20\ \mathrm{lg}\frac{U_{2\,\mathrm{RMS}}}{U_{1\,\mathrm{RMS}}}\right)\mathrm{dB}$$

These quantities used to be called field quantities, but this designation was misleading. Power and energy densities are both field and power quantities. Electric voltage and electric current are not field quantities but integrals over field quantities. Therefore, ISO 80000-1 [3] introduced the designation "root power quantity".

This designation is likely to be adopted in the next versions of the IEC 60027 standards.

Logarithmic ratios of quantities, level

The logarithmic ratio of two electrical quantities is defined as a level when the denominator is the fixed value of a reference quantity of the same dimension as the numerator [9]. The unit used is the decibel (dB). The value of the reference quantity should always be specified for numerical values of levels. To abbreviate this specification, the reference quantity in parentheses can follow the dB symbol. If the numerical value of the reference quantity equals 1, it can be omitted. To make clear that this is not a special unit but only a reference value, a space should separate the dB symbol and the expression in parentheses (see [9]). Reference [9] also mentions some abbreviations introduced by the International Telecommunication Union (ITU) [10]. In these abbreviations, dB is directly followed by a letter or a sequence of characters to identify the reference value. IEC 60027-3 ([9]) recommends not to use these abbreviations.

Definition for power quantities

Example: power P, reference value P_0

$$L_P(\text{re } P_0) = L_{P/P_0} = 10 \text{ lg}\frac{P}{P_0} \text{ dB}$$

Definition for quantities whose square is proportional to a power quantity

Example: voltage U, reference value U_0

$$L_U(\text{re } U_0) = L_{U/U_0} = 20 \text{ lg}\frac{U}{U_0} \text{ dB}$$

Table 5 contains some level definitions and short forms. Other level definitions that are common in telecommunications are listed in IEC 60027-2 [8].

The difference between two levels is an attenuation or gain figure. Adding an attenuation or gain figure to a level again yields a level. This mathematical operation only complies with the rules of algebra if both the level and the attenuation or gain figure are specified in dB without any additional information. Therefore, the short forms in columns 5 and 6 of Table 5 are only suitable for indicating measured values and results. In general, signs denoting reference values and measurement methods should be appended to the quantity symbol and not to the unit. This applies not only to the SI units but also to the decibel.

Relation between electric and magnetic field strength levels

The field strengths are linked by the equation $E_{\mathrm{RMS}} = Z_{\mathrm{F}} \cdot H_{\mathrm{RMS}}$, where Z_{F} is the field characteristic impedance.

Conversion into level:

$$20\ \lg\left(\frac{E_{\mathrm{RMS}}}{1\ \mu\mathrm{V/m}}\right)\ \mathrm{dB} = 20\ \lg\left(\frac{H_{\mathrm{RMS}}}{1\ \mu\mathrm{A/m}} \cdot \frac{Z_{\mathrm{F}}}{1\ \Omega}\right)\ \mathrm{dB}$$

$$= 20\ \lg\left(\frac{H_{\mathrm{RMS}}}{1\ \mu\mathrm{A/m}}\right)\ \mathrm{dB} + 20\ \lg\left(\frac{Z_{\mathrm{F}}}{1\ \Omega}\right)\ \mathrm{dB}$$

The expression

$$A_{Z/\Omega} = 20\ \lg\left(\frac{Z_{\mathrm{F}}}{1\ \Omega}\right)\ \mathrm{dB}$$

and the letter symbols in Table 5 can be used to describe the relation between the field strength levels as follows:

$$L_{E/(\mu\mathrm{V/m})} = L_{H/(\mu\mathrm{A/m})} + A_{Z/\Omega}$$

where $A_{Z/\Omega}$ is the impedance conversion figure (suggested designation). This is not a level because the reference value is neither a power quantity nor a root power quantity. dB (Ω) can be used as the short form. dBΩ should be avoided.

Table 5: Examples of level definitions with different reference quantities

Quantity, reference value	Letter symbol		Level, definition	Unit, short form	
	Long form	Short form		IEC	ITU
Electric power, reference value: 1 mW	L_P (re 1 mW)	$L_{P/\mathrm{mW}}$	$10\ \lg\left(\frac{P}{1\ \mathrm{mW}}\right)\ \mathrm{dB}$	dB (mW)	dBm
Voltage, electric tension, reference value: 1 V	L_U (re 1 V)	$L_{U/\mathrm{V}}$	$20\ \lg\left(\frac{U_{\mathrm{RMS}}}{1\ \mathrm{V}}\right)\ \mathrm{dB}$	dB (V)	dBV
Voltage, electric tension, reference value: 1 µV	L_U (re 1 µV)	$L_{U/\mu\mathrm{V}}$	$20\ \lg\left(\frac{U_{\mathrm{RMS}}}{1\ \mu\mathrm{V}}\right)\ \mathrm{dB}$	dB (µV)	dBµV
Electric field strength, reference value: 1 µV/m	L_E (re 1 µV/m)	$L_{E/(\mu\mathrm{V/m})}$	$20\ \lg\left(\frac{E_{\mathrm{RMS}}}{1\ \mu\mathrm{V/m}}\right)\ \mathrm{dB}$	dB (µV/m)	not: dBµV/m
Magnetic field strength, reference value: 1 µA/m	L_H (re 1 µA/m)	$L_{H/(\mu\mathrm{A/m})}$	$20\ \lg\left(\frac{H_{\mathrm{RMS}}}{1\ \mu\mathrm{A/m}}\right)\ \mathrm{dB}$	dB (µA/m)	not: dBµA/m
Relative noise level Carrier power: P_{c} Spurious signal power: P_{n}	L_{n} (re P_{c})	$L_{\mathrm{n},P_{\mathrm{c}}}$	$10\ \lg\left(\frac{P_{\mathrm{n}}}{P_{\mathrm{c}}}\right)\ \mathrm{dB}$	dB (P_{c})	dBc

Notation

The notation for quantities and units is standardized internationally in [3]; see also [2].

Italics

The following are written in italic (sloping) type:

- Letter symbols for physical quantities, e.g. m (mass); U (voltage)
- Letter symbols for variables, e.g. x; n
- Symbols for functions and operators with user-definable meaning, e.g. $f(x)$

A serif font (e.g. Times) should be used; see also [6].

Roman type

The following are written in roman (upright) type:

- Units and their prefixes, e.g. m; kg; s; pF; V; dB
- Numerals, e.g. 4.5; 67; 8-fold; 1/2
- Symbols for functions and operators with fixed meaning, e.g. sin; lg; π
- Chemical elements and compounds, e.g. Cu; $\mathrm{H_2O}$

The author recommends using a sans serif font (e.g. Arial).

These rules of notation allow a clear distinction between quantity symbols and unit symbols.

Quantity values in tables and diagrams

Tables 6 and 7 show examples of correct and wrong labeling of table headers and coordinate systems.

The labeling used for the displays of test and measurement instruments should also follow these recommendations. The extensive functionality of state-of-the-art electronic test and measurement instruments often causes problems because space and character set are limited. Therefore, it is sometimes necessary to make compromises.

Table 6: Labeling of table headers and coordinate systems

Correct					**Wrong** [3]		
U	U/V	U in V	E/(V/m)	E in V/m	U [V]	U [V]	U in [V]
0.1 V	0.1	0.1	0.1	0.1	0.1	0.1	0.1
0.2 V	0.2	0.2	0.2	0.2	0.2	0.2	0.2
…	…	…	…	…	…	…	…

[3] Do not put units in brackets.

Table 7: Labeling of table headers and coordinate systems for large value ranges

Correct			**Wrong** [4]
P/W	P/W	P	P/W
1	1	1 W	1
$1 \cdot 10^{-3}$	10^{-3}	1 mW	1 m
$1 \cdot 10^{-6}$	10^{-6}	1 µW	1 µ
$1 \cdot 10^{-9}$	10^{-9}	1 nW	1 n

[4] Do not use prefixes alone.

Frequent mistakes

Many articles in technical journals, documentation and papers do not comply with the correct usage of quantities, units and equations as specified by the relevant national and international standards.

Especially in electrical engineering, it is common bad practice to add indices to units. This practice violates the relevant standards. An index must always be appended to the quantity symbol, not to the unit symbol. As a result of this incorrect usage, units are converted, although conversions of quantities are referred to. In this context, the decibel (dB) causes particular problems. All these problems can be avoided if quantities are defined instead of special units and the reference value is appended to the quantity symbol as an index.

Another case of non-compliance with standards is placing the unit in brackets next to the quantity symbol. This bad practice is unfortunately very common. A scaled quantity equation is recommended when the unit as well as the quantity is to be given.

It is strongly recommended that the checklist in [2] be used when revising manuscripts.

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References

No.	Title
[1]	Le Système international d'unités/The International System of Units (8th edition, 2006) (www.bipm.fr).
[2]	Thompson, A., Taylor, B.N.: Guide for the Use of the International System of Units (SI), Gaithersburg, MD20899, 2008 (NIST publication).
[3]	ISO 80000-1: 2009, Quantities and units – Part 1: General.
[4]	ISO 80000-6: 2008, Quantities and units – Part 6: Electromagnetism.
[5]	International vocabulary of metrology – Basic and general concepts and associated terms (VIM), 3rd edition JCGM 200: 2008 (E/F) (www.bipm.org/utils/common/documents/jcgm/JCGM_200_2008.pdf).
[6]	IEC 60050-112: 2010, IEV PART 112: QUANTITIES AND UNITS (E/F) (www.electropedia.org).
[7]	IEC 60027-1: 1992, Letter symbols to be used in electrical technology – Part 1: General.
[8]	IEC 60027-2: 2000, Letter symbols to be used in electrical technology – Part 2: Telecommunications and electronics.
[9]	IEC 60027-3: 2004, Letter symbols to be used in electrical technology – Part 3: Logarithmic and related quantities, and their units.
[10]	ITU-R V.574-4: 2000, Use of the decibel and the neper in telecommunications.

Designations of the cited institutions

- JCGM: Joint Committee for Guides in Metrology
- BIPM: International Bureau of Weights and Measures
- ISO: International Organization for Standardization
- IEC: International Electrotechnical Commission
- CENELEC: Comité Européen de Normalisation Electrotechnique

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PD 5214.5061.62 | Version 03.00 | September 2012 |
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5214506162